Managing Resources in Multi-resource General Lotto Games

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System planners have the responsibility to allocate limited resources towards accomplishing multiple objectives. Optimal allocation strategies are increasingly becoming more complex, given the need to utilize multiple heterogeneous types of resources that are specialized for completing particular tasks. For example, multirobot systems require the allocation and coordination of agents with heterogeneous capabilities in order to carry out a variety of tasks [1], [2], [3]. In defense and security scenarios, ensuring the security of a power grid requires the allocation of cyber-security software, physical security infrastructures, and human resources [4], [5], [6], [7], [8]. As such, the successful completion of multiple tasks and objectives often cannot be accomplished through the allocation of only a single resource type. This poses new challenges as to how strategies for the allocation of heterogeneous resources should be conceived. Indeed, a system planner seeks to maximize performance by utilizing multiple types of resources with different capabilities, effectiveness, and costs.

In this paper, we focus on deriving the optimal allocation of multiple, heterogeneous resources types in strategic adversarial environments. In particular, we propose an extended formulation of the General Lotto game, a popular model of competitive resource allocation between opponents [9], [10]. The General Lotto game is a variant of the classic Colonel Blotto game [11]. Two budget-constrained players compete over a set of valuable contests, where the player that deploys more resources to a particular contest wins an associated value. In the standard formulations, the players have access only to a single resource type, and the winner of each contest is determined purely by the amount of resources allocated to that contest – this is known as the winner-take-all winning rule. In this paper, we consider formulations where players have access to multiple resource types. Now, the winner of an individual contest depends on the multi-dimensional composition of resource types that are allocated. Thus, the specification of the winning rules in this formulation is an important and novel modeling consideration.

The literature on General Lotto and Colonel Blotto games primarily considers the allocation of a single resource type. As such, the study of multiple, heterogeneous resource types is largely unexplored. Single-resource models have studied many other aspects of adversarial interactions such as multi-agent settings [12], [13], incomplete information [14], [15], and networked settings [16], [17]. Few studies have considered the multiple and heterogeneous resource types. A multi-resource integer Blotto game was considered in [18], which focused on deriving efficient computational algorithms rather than studying specific winning rules and the corresponding equilibrium payoffs. Recent interest in defense applications proposes a framework for "mosaic warfare", where multiple resource types have heterogeneous capabilities and effectiveness against the opponent's resource types [19]. The effect of fractionated resource types (quantized amounts of resources) has also been considered [20]. These works primarily provide computational insights. An analysis of tradeoffs between two types of resources that are allocated in different time periods, i.e. pre-allocated and real-time resource types, is recently given in [21], [22] for General Lotto games.

In this paper, we propose a framework to study classes of multi-resource General Lotto games, which is an extension of the General Lotto game to scenarios where players have two resource types at their disposal. Here, we seek to provide analytical equilibrium characterizations. In particular, our main contributions here are the derivation of equilibrium payoffs and strategies under a weakest-link/best-shot winning rule for the individual contests for two resource types. That is, one of the players must send more resources of *every* type in order to win a battlefield (weakest-link), while the other player only needs to have more of any one resource type (best-shot). Additionally, we provide an equilibrium characterization when the resource types have heterogeneous effectiveness on the battlefields, i.e. the winning rule depends on a linear combination of the allocated resource types.

In both winning rules, weakest-link and linear combination, the equilibrium payoffs are a function of $\gamma(\cdot)$, where $\gamma(v)$ is equal to $1 - \frac{v}{2}$ for v less than one and is equal to $\frac{1}{2v}$, otherwise. However, the γ function has a breaking point at $v = 1$, it is a differentiable function. The breaking point exists because players choose different strategies based on their budgets. In each winning rule, the breaking point is a function of the budget value for each type of resource. In the weakest-link winning rule, the breaking point is the summation of the ratio of each type of budget, while in the linear combination winning rule, it is the ratio of the summation of each type of budget.

We then leverage the equilibrium solutions to fixed-budget settings to study the problem of investing in resource types that are costly. More precisely, we focus on a two-stage interaction where each player has a limited monetary budget for which to invest in the two resource types before competition takes place. For each player, each resource type has a per-unit cost, to invest in. The interaction unfolds as follows.

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– Stage 1: Both players decide how much of each resource type to purchase given their monetary budgets.
– Stage 2: The players engage in a multi-resource General Lotto game (e.g. with weakest-link or linear winning rules), and the subsequent payoffs from the multi-resource game are derived.

For this problem, we derive the equilibrium investment in each resource type. Again, here, the equilibrium payoffs are a function of $\gamma(\cdot)$. In weakest-link winning rule, the breaking point depends on the summation of the ratio of costs, while in linear combination winning rule, it depends on the ratio of the summation of the costs. Here, we emphasize the multi-dimensional aspect of our formulation by showing a fixed performance level can be achieved by a contour line of resource budgets.

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